$$\theta^n - 1 = (\theta - 1)(\theta^{n-1} + \theta^{n-2} + \ldots + 1),\ \dot{\theta}/\theta = c_2 - c_1,$$

we arrive at the following estimate for the sign: $(\theta^n - 1)\dot{\theta}/\theta \geqslant 0$ for $t \in [0,\ (1/2)(1 - \gamma^{-1})]$ and $t \in [(1 - \gamma^{-1}),\ 1]$. In the intermediate interval $t \in ((1/2)(1 - \gamma^{-1}),\ 1 - \gamma^{-1})$ the expression under consideration is negative. In this case the general estimate of the sign of W depends on the relationship between the two members in (4.1). It is important to note that the dissipative function is positive for any $\gamma > 0$ in the initial state of the strain of a viscoplastic strip.

LITERATURE CITED

1. A. A. Il'yushin, "Strain of a viscoplastic body," Uch. Zap. Moscow State Univ., Mekhanika, No. 39 (1940).
2. A. Yu. Ishlinskii, "On stability of viscoplastic flow of a strip and circular rod," Prikl. Mat. Mech., 7, No. 2 (1943).
3. G. D. Del', "Stability of biaxial tension of a sheet of viscoplastic material," in: Mechanics of Deformed Bodies and Structures [in Russian], Mashinostroenie, Moscow (1975).
4. V. M. Kuznetsov and E. N. Sher, "On the stability of ideal fluid flow in a strip and a ring," Prikl. Mekh. Tekh. Fiz., No. 2 (1964).
5. A. G. Ivanov, "Features of explosive strain and rupture of tubes," Probl. Prochn., No. 11 (1976).
6. Yu. I. Fadeenko, "Time criterion of rupture in solid body dynamics," in: Dynamics of a Continuous Medium [in Russian], No. 32 Izd. Inst. Gidrodinam. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1977).
7. O. A. Baum, L. P. Orlenko, et al., Physics of Explosion [in Russian], Nauka, Moscow (1975).
8. S. K. Godunov, A. A. Deribas, et al., "Investigation of the viscosity of metals in high-velocity collisions," Fiz. Goreniya Vzryva, 7, No. 1 (1971).
9. Yu. I. Fadeenko, "Time criteria of rupture by explosion," Prikl. Mekh. Tekh. Fiz., No. 6 (1977).
10. G. P. Cherepanov, Mechanics of Brittle Fracture [in Russian], Nauka, Moscow (1974).
11. D. D. Ivlev, Theory of Ideal Plasticity [in Russian], Nauka, Moscow (1966).

INVESTIGATION OF MATERIAL DAMAGE UNDER CREEP AND CREEP STRENGTH

A. M. Lokoshchenko

UDC 539.4:539.376

Within the framework of the mechanics of continuous media, the conception of the mechanical equation of state with a system of kinetic equations to determine the parameters q_i characterizing the state under consideration

$$\dot{p} = \dot{p}(\sigma,\ T,\ q_1,\ q_2,\ \ldots,\ q_n); \tag{1}$$

$$dq_j = a_j d\sigma + b_j dT + c_j dt,\ j = 1,\ 2,\ \ldots,\ n. \tag{2}$$

which has been proposed in [1], is often the starting point to describe metal creep. According to (1), the creep rate is determined by the stress σ, the temperature T, and a certain number of structural parameters q_j. In the general case, (2) represent nonintegrable kinetic relationships to describe changes in the parameters q_j, which in turn characterize a change in the material structure (a_j, b_j, c_j are certain functions of σ, T, t, as well as of q_j). In order to describe at least certain qualitative features of the creep strength of metals, one structural parameter ω is most often introduced for simplicity, and it is taken as a certain measure of material damage. In solving creep and creep strength problems, usually either the physical meaning of the parameter ω is not made specific, or ω is understood to be the relative part of the specimen section damaged as a result of creep. The rupture time $t = t^*$ is often understood to be the time at which the damage reaches unity ($\omega^* = \omega(t^*) = 1$). In

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 129-133, November-December, 1982. Original article submitted November 3, 1981.

0021-8944/82/2306-0855$07.50

TABLE 1

Specimen No.	σ_0	t^*	ω^*
2(I)	40	68,0	0,339
3(I)	40	66,5	0,534
4(I)	50	40,0	0,207
5(I)	50	37,0	0,252
7(I)	60	24,0	0,126
8(I)	60	28,0	0,371
10(I)	70	11,75	0,177
11(I)	70	11,5	0,039

this paper we present results of an experimental investigation of $\omega(t)$; it is shown that $\omega^* < 1$, where the dependence of ω^* on the stress is monotonically decreasing in nature; an analytic description of this effect is obtained.

The specimens were tested under conditions of a constant tensile force acting at a temperature of T = 400°C. All the specimens were fabricated from one mark M3 copper rod. In all 21 specimens were fabricated, 12 in series I (with constant cross section), and 9 specimens of series II (with piecewise constant section). Before the tests all the specimens were subjected to simultaneous annealing in a vacuum ($1.3 \cdot 10^{-3}$ Pa) for two hours at a temperature of 800°C, and subsequent cooling with the furnace. Everywhere below σ_0 is understood to be the conditional stress, equal to the ratio between the tensile force and the area of the undeformed section. The material under consideration is characterized by substantial nonlinearity of the instantaneous properties at T = 400°C, the yield point is $\sigma_{0S} \cong 30$ MPa, the strength is $\sigma_{0B} = 120$ MPa. The tests were performed at the stresses $\sigma_0 =$ 40; 50; 60; 70 MPa.

Specimens of series I (with a 50-mm working length and from 4.27-5.65-mm initial diameter) were tested in a IMekh-5 apparatus, supplemented with a control shield for automatic regulation and a temperature record from a thermocouple mounted on the specimen. The temperature was maintained constant with ±3°C accuracy. Results of testing these specimens are presented in Table 1 (the Roman numbers in parentheses are the specimen series). The values of σ_0 in all the tables are presented in MPa and τ^* in h.

Specimens of series II have two, three, or four steps along the generatrix. Specimens 1(II)-3(II) are characterized by two parts of constant section, specimens 7(II)-9(II) by three parts, specimens 4(II)-6(II) by four parts. The magnitudes of the diameters of the different steps of specimens of the series II are selected in such a manner that the stress σ_0 under a 100 kg tensile force would equal one of the values $\sigma_0 = 40$; 50; 60; 70 MPa at each step. The total working length of each specimen of series II is 80 mm since the working lengths of the different steps varied between 20 and 40 mm. Creep strength tests of specimens of series II were performed on the multicalibrating apparatus DR-9 [2], supplemented by a high-temperature furnace. Chains of three successively connected specimens were tested. The construction of the apparatus was such that rupture of certain specimens in the chain does not hinder continuing the testing of the remaining specimens in the same chain. Checking thermocouples were fastened directly to the specimen, and regulating thermocouples to the outside of catches in the specimen domain. Specimens in the upper (1(II)-3(II)) and middle (7(II)-9(II)) zones of the furnace were tested at the 400°C temperature to ±3°C accuracy; the mean temperature of specimens 4(II)-6(II) in the lower zone of the furnace was 385°C. In this connection, the rupture time for specimens 4(II)-6(II) was somewhat higher than for the series I specimens 10(I)-11(I) tested at the same stress $\sigma_0 = 70$ MPa. The rupture times of all the series II specimens, tested at T = 400°C, are in good agreement with the rupture times of the series I specimens tested for the same values of T and σ_0. The results of experiments on the series II specimens are presented in Table 2. The parameter i is issued to characterize the number of specimen steps, the greater value of the step diameter and the correspondingly smaller value of the stress σ_0 correspond to the large value of i. In this connection rupture always sets in on the step I (i = 1).

The mean values of t^* obtained from test results of all specimens at T = 400°C in both installations are presented in Table 3. The creep strength characteristics of the material being used are approximated well by a straight line in the semilogarithmic coordinates (σ_0—log t^*), and by a broken line with breakpoint within the stress range under considera-

TABLE 2

Specimen number	T°, C	t*	i=1			i=2			i=3			i=4		
			σ_0	τ	ω	σ_0	τ	ω	σ_0	τ	ω	σ_0	τ	ω
1(II)		42			0,485			0,229						
2(II)	400	40	50	1	0,503	40	0,62	0,324	—	—	—	—	—	—
3(II)		43			0,376			0,108						
4(II)		19			0,222			0,061			0,053			0,043
5(II)	385	23	70	<1,81	0,230	60	<0,90	0,168	50	<0,52	0,054	40	<0,31	0,018
6(II)		21			0,145			0,088			0,046			0,033
7(II)		20			0,201			0,090			0,052			
8(II)	400	25	60	1	0,408	50	0,55	0,083	40	0,33	0,090	—	—	—
9(II)		21			0,209			0,122			0,004			

TABLE 3

σ_0	t*	ω*
40	67,2	0,44
50	40,4	0,36
60	23,7	0,26
70	11,6	0,11

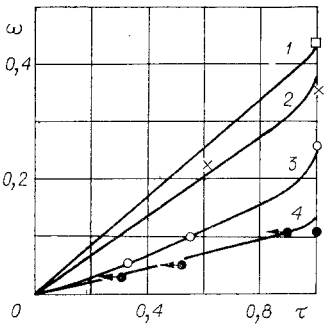

Fig. 1

tion (at log σ_0—log t^*) in double logarithmic coordinates σ_0 = 56 MPa. A value of the dimensionless time τ, equal to the ratio between the real time for the stress σ_0 to act on a given step and the time $t^*(\sigma_0)$ during which the material can sustain this stress, is presented in Table 2 for each step of the specimens.

Metallographic investigation of the specimens was performed on microsections cut from the midsections of each step along the axial direction. Etching the microsections was performed in a reagent of the following composition: 50 ml water, 50 ml ammonia, and 5 ml perhydrol. In the initial state after annealing, the specimens have the structure of equiaxial grains without pores and microcracks. The mean grain diameter D of the initial structure is 0.096 mm. Microcracks, and in certain cases pores also, were detected in large quantities on the microsections of the series I specimens as well as on the microsections of the ruptured steps (i = 1) of the series II specimens. The microcracks were located along the grain boundaries, they were hence mainly perpendicular to the direction of the applied stress. In practice there were no intergranular cracks. The microcrack size was considerably smaller in the unruptured steps (i = 1) of the series II specimens than in the first step. A characteristic feature of the unruptured step microstructure is the presence of a large quantity of micropores on the grain boundaries.

A transverse path in the shape of a rectangle, one of whose sides would agree with the specimen diameter d and the other (along the specimen axis) with an arbitrary dimension H, was selected for the quantitative determination of the material damage in each microsection. The sum of the lengths of all the intergranular boundaries perpendicular to the tension direction was Hd/D (where D is the specimen grain size). Let us calculate the quantity a, the sum of lengths of projections of all microcracks in the direction of the specimen diameter.

We take as a measure of the damage ω the ratio between the total length of the transverse boundaries occupied by the pores and microcracks, and the length of all the transverse boundaries between the grains $\omega = aD/Hd$. Since the microsections indicate a uniform microcrack and pore distribution along the radial coordinate of the microsections, then the relative fracturing ω of the specimen diameter determined by the above-mentioned method agrees with the relative fracturing of the circular cross-sectional area. Values of the damage ω are presented in Tables 1 and 2 for all steps of the specimens tested, while the mean values of the damage ω^*, corresponding to the time of rupture, are in Table 3. Superposed in the figure are curves of the dependence of the damage ω on the relative time τ of the specimen under load (lines 1-4 correspond to σ_0 = 40; 50; 60; 70 MPa). Table 3 and the figure show that the dependence ω^* is monotonically decreasing in nature. From the viewpoint of the material structure such a phenomenon can be explained as follows. For small stresses, the creep is accompanied by the development of a pore and crack formation process along the grain boundaries and a subsequent intergranular rupture. At high stresses, rupture sets in because of the development of irreversible creep shear strains that are apparently associated with the creeping over dislocations, the quantity of pores and cracks here is relatively small.

Let us turn to a description of this phenomenon by using a phenomenological approach. The strain ε under an instantaneous loading is associated in [3] with the effective stress s dependent on the damage parameter ω:

$$\varepsilon = G(s), \quad \omega = g(s),$$

and the creep process is determined by a system of two differential equations

$$\dot{\varepsilon} = G'(s)\dot{s} + F(s), \quad \dot{\omega} = g'(s)\dot{s} + f(s), \tag{3}$$

where the point (as above) denotes differentiation with respect to time t and the prime with respect to the effective stress s. The functions G'(s), F(s), g'(s) and f(s) in (3) will be considered continuous, and monotonically increasing from the origin. Exactly as in [4], we shall use the following definitions of ε and ω

$$\varepsilon = \ln (l/l_0) = \ln (A_0/A), \quad \omega = \ln [A/(A - A_\omega)], \tag{4}$$

where l_0 and A_0 are the length and cross-sectional area of the specimen prior to load application, l and A are analogous quantities during deformation, and A_ω is the mean area of voids of a different type in the section. According to (4), the effective stress for tension on a specimen by a constant force P is defined as follows:

$$s = P/(A - A_\omega) = \sigma_0 \exp (\varepsilon + \omega), \quad \sigma_0 = P/A_0. \tag{5}$$

At the beginning of the creep process (for t = +0), an effective stress s_0 defined by using (2) and (5) as

$$s_0 = \sigma_0 \exp (G_0 + g_0), \quad G_0 = G(s_0), \quad g_0 = g(s_0). \tag{6}$$

occurs in the specimen because of instantaneous application of the force P. A dependence of the effective stress s on the time t is obtained in [4], which can be used to determine the value of s* at the time of rupture from the condition $\dot{s} \to +\infty$:

$$\frac{1}{s^*} - G'^* - g'^* = \frac{1}{s^*} - \frac{dG}{ds}\Big|_{s=s^*} - \frac{dg}{ds}\Big|_{s=s^*} = 0. \tag{7}$$

Let us analyze this condition. According to (7), the limit value of the effective stress s* is determined only by the kind of functions G(s) and g(s) that characterize the material behavior under an instantaneous loading, and is independent of the magnitude of the nominal stress σ_0. By using (3) and (5), the value corresponding to the time of rupture $\omega^* = \omega(t^*)$ can be determined

$$\omega^* = g^* + \int_{s_0}^{s^*} \frac{(1/s - G' - g')f}{(F + f)}\, ds. \tag{8}$$

Let us consider the dependence of the limit value of the damage ω^* on the stress σ_0. Only the lower limit of the integrals in (8) depends on σ_0, where $s_0(\sigma_0)$ is a growing function. All the remaining quantities in (8) are determined by the material properties and are independent of σ_0. Hence, increasing the stress σ_0 results in diminishing the limit value of the damage ω^* corresponding to the time of specimen rupture. Therefore, the model (3) per-

mits description of the monotonic decrease in the limit damage ω^* due to the applied stress σ_0, which is obtained from a metallographic analysis.

Let us examine the case of small strains ($\varepsilon \ll 1$). We introduce the most convenient form for the functions g(s):

$$g(s) = Cs^m. \tag{9}$$

We determine the form of the monotonically decreasing dependence $\omega^*(\sigma_0)$ in this case. Since the effective stress s under small strains differs from the nominal stress σ_0 only because of the cumulative damage ω, then (5) takes the following form in this case

$$s = \sigma_0 \exp \omega. \tag{10}$$

Using (7) and (10), we obtain

$$\omega^* = \ln (D_1/\sigma_0), \; D_1 = (Cm)^{-1/m}.$$

We consider the case of small strains for another definition of the effective stress

$$s = \sigma_0/(1 - \omega). \tag{11}$$

In this case (3) and (11) result in the following dependence $\dot{\omega}(t)$:

$$\dot{s} = \frac{\sigma_0 \dot{\omega}}{(1-\omega)^2}, \qquad \dot{\omega} = \frac{f}{\left[1 - \dfrac{\sigma_0 g'}{(1-\omega)^2}\right]}. \tag{12}$$

In conformity with (12), the limit value ω^* for the power-law function (9) has the form

$$\omega^* = 1 - D_2 \sigma_0^{m/(m+1)}, \quad D_2 = (Cm)^{1/(m+1)}. \tag{13}$$

Therefore, in the case of small strains the definition of the effective stress in the form (10) or (11) results in a monotonic decrease of $\omega^*(\sigma_0)$ in the form of a logarithmic or a power-law dependence, respectively. It follows from (13) that $0 < \omega^* < 1$, i.e., rupture always sets in during creep when the corss-section is not filled completely by cracks.

The author is grateful to T. M. Aver'yanova, L. I. Gryaznova, and V. I. Nikolaev for assistance in performing the experiments.

LITERATURE CITED

1. Yu. N. Rabotnov, Creep of Structure Elements [in Russian], Nauka, Moscow (1966).
2. E. A. Myakotin and S. A. Shesterikov, "Nine tubular specimen installation to investigate the creep strength of metals in the complex stress state," in: Deformation and Rupture of Solids [in Russian], Moscow State Univ., Moscow (1977).
3. H. Broberg, "A new criterion for brittle creep rupture," Trans. ASME, E41, No. 3 (1974).
4. P. O. Boström, H. Broberg, L. Bräthe, and M. Chrzanowski, "On failure conditions in viscoelastic media and structures," in: Int. Symp. on Mechanics of Viscoelastic Media and Bodies, Gothenburg, Springer-Verlag, Berlin (1975).